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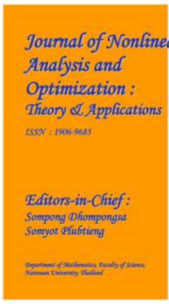

INTUITIONISTIC FUZZY q- IDEALS OF BCI – ALGEBRAS WITH DEGREES IN THE INTERVAL (0, 1]

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Abstract

The notion of an enlarged q-ideal and an Intuitionistic fuzzy q-ideal in BCI-algebras with degree are introduced. Related properties of them are investigated.

Key Words: Enlarged q-ideal, Intuitionistic Fuzzy q-Ideals with degree.
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1. Introduction

BCK- algebras and BCI-algebras are two classes of logical algebras, which were initiated by K. Iseki[5,6]. The notion of fuzzy sets, invented by L.A. Zadeh[11], has been applied in many field. In 1991, O.G. Xi[10] applied it to BCK-algebras. Since then fuzzy BCK/BCI-algebras have been extensively investigated by several researchers. The concept of a fuzzy set is applied to generalize some of the basic concepts of general topology[3]. Rosenfeld [9] constituted a similar application to the elementary theory of groupoids and groups. Xi[10] applied to the concept of fuzzy set to BCK-algebras. Y. L.Liu et al.[8] defined the notions of q-ideals and a-ideals in BCI-algebras and studied their properties. The idea of "Intuitionistic fuzzy set" was first published by Atanassov [1,2] as a generalization of the notion of fuzzy sets. After that many researchers considered the Intuitionistic fuzzification of ideas and subalgebras in BCI/BCK – algebras. The aim of this paper is to introduce the notion of an enlarged q-ideals and Intuitionistic fuzzy q-ideals in BCI- algebras with degree and study related properties of them.

2. Preliminaries

By a BCI-algebra we mean an algebra $(X,*,0)$ of type (2,0) satisfying the following conditions:
(a1) $(\forall x,y,z \in X)\ \Big(\big((x*y)*(x*z)\big)*(z*y)=0\Big)$,
(a2) $(\forall x,y \in X)\ \Big(\big(x*(x*y)\big)*y=0\Big)$,
(a3) $(\forall\ x \in X)\ (x*x=0)$,
(a4) $(\forall x,y \in X)\ (x*y=0, y*x=0) => x=y)$.

A BCI-algebra X is called a BCK-algebra if it satisfies the following identity:
(a5) $(\forall x \in X)\ (0*x)=0$
In any BCI-algebra X one can define a partial order " $\leq$ " by putting $x \leq y$ if and only if $x*y=0$.

A BCI-algebra X has the following properties:
(b1) $(\forall x \in X)\ (x*0=x)$.
(b2) $(\forall x,y,z \in X)\ \big((x*y)*z=(x*z)*y\big)$.
(b3) $(\forall x,y \in X)\ \big(0*(x*y)=(0*x)*(0*y)\big)$.
(b4) $(\forall x,y \in X)\ \big(x*\big(x*(x*y)\big)=x*y\big)$.

(b5) $(\forall x, y, z \in X)\ (x \leq y\ \ =>\ \ x * z \leq y * z,\ \ z * y \leq z * x)$.
(b6) $(\forall x, y, z \in X)\ (x * z) * (y * z) \leq x * y)$.
(b7) $(\forall x, y, z \in X)\ \left(0 * \left(0 * \left((x * z) * (y * z)\right)\right) \leq (0 * y) * (0 * x)\right)$.
(b8) $(\forall x, y \in X)\ (0 * (0 * (x * y)) = (0 * y) * (0 * x))$.

A non-empty sub-set S of a BCI-algebra X is called a subalgebra of X if $x * y \in S$ whenever $x, y \in S$. A non-empty subset A of a BCI-algebra X is called an *ideal* of X if it satisfies:
(c1) $0 \in A$,
(c2) $(\forall x \in A)(\forall y \in X)(y * x \in A\ \ => y \in A)$.
Note that every ideal A of a BCI-algebra X satisfies:
$(\forall\ x \in A)(\forall y \in X)(y \leq x\ \ => y \in A)$.
A non-empty subset A of a BCI-algebra X is called a *q-ideal* ([8]) of X if it satisfies (c1) and
(c3) $(\forall x, y, z \in X)(x * (y * z) \in A\ \ and\ \ y \in A\ \ => x * z \in A)$.
Any q-ideal is an ideal, but the converse is not true in general.

Definition 2.1: A fuzzy subset μ of a BCK/BCI-algebra X is called a *fuzzy ideal* ([7]) of X if it satisfies:
(d1) $(\forall x \in X)\big(\mu(0) \geq \mu(x)\big)$,
(d2) $(\forall x, y \in X)(\mu(x) \geq \min\{\mu(x * y), \mu(y)\})$.

Definition 2.2 : An Intuitionistic fuzzy set A in a non-empty set X is an object having the form $A = \{(x,\ \mu_A(\mathrm{x}),\ \gamma_A(\mathrm{x}))/\mathrm{x} \in \mathrm{X}\}$, where the functions $\mu_A\!: \mathrm{X} \to [0,\ 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership $\big(namely\ \gamma_A(x)\big)\ \ of\ each\ element\ x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1\ for\ all\ x \in X$.

Such defined objects are studied by many authors (see example two journals: 1. Fuzzy sets and 2. Notes on Intuitionistic Fuzzy Sets) have many interesting applications not only in mathematics (See Chapter 5 in the book [2]).

For the sack of simplicity , we shall use the symbol $A = (\mu_A, \gamma_A)$ for the intuitionistic fuzzy set $A = \{(x,\ \mu_A(\mathrm{x}),\ \gamma_A(\mathrm{x}))/\mathrm{x} \in \mathrm{X}\}$.

Definition 2.3: An Intuitionistic fuzzy subset $A = (\mu_A, \gamma_A)$ of a BCK/BCI-algebra X is called an *Intuitionistic fuzzy ideal* of X if it satisfies:
(i1) $(\forall x \in X)\big(\mu_A(0) \geq \mu_A(x)\big),\ \big(\gamma_A(0) \leq \gamma_A(x)\big)$,
(i2) $(\forall x, y \in X)(\mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\})$,
(i3) $(\forall x, y \in X)(\gamma_A(x) \leq \max\{\gamma_A(x * y), \gamma_A(y)\})$.

Proposition 2.4: If $A = (\mu_A, \gamma_A)$ is an Intuitionistic fuzzy ideal of a BCI-algebra X, then the following holds:
$(\forall x, y \in X)(x \leq y\ \ =>\ \ \mu_A(x) \geq \mu_A(y)$ and $\gamma_A(x) \leq \gamma_A(y))$.

3. Intuitionistic Fuzzy q-ideals of BCI-Algebras

Definition 3.1: An Intuitionistic fuzzy subset $A = (\mu_A, \gamma_A)$ of a BCI-algebra X is called an *Intuitionistic fuzzy q-ideal* of X if it satisfies (i1) and
(i4) $(\forall x, y, z \in X)(\mu_A(x * z) \geq \min\{\mu_A(x * (y * z), \mu_A(y)\})$,
(i5) $(\forall x, y, z \in X)(\gamma_A(x * z) \leq \max\{\gamma_A(x * (y * z), \gamma_A(y)\})$.

Example 3.2: Let $X = \{0, a, b, c\}$ be a BCI-algebra in which the $*$ - operation is given by the following table:

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Note that {0,a} is a q-ideal of X. Define an Intuitionistic fuzzy subset μ_A: X $\rightarrow [0,\ 1]$ and $\gamma_A : X \rightarrow [0, 1]$ by

$\mu_A = \begin{pmatrix} 0 & a & b & c \\ 0.8 & 0.7 & 0.5 & 0.5 \end{pmatrix}$ and $\gamma_A = \begin{pmatrix} 0 & a & b & c \\ 0.2 & 0.2 & 0.4 & 0.3 \end{pmatrix}$

Then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy q-ideal of X.

Proposition 3.3: Every Intuitionistic fuzzy q-ideal of a BCI-algebra X is an intuitionistic fuzzy ideal of X.

Proof: Let $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy q-ideal of X. Let $x, y \in X$. Putting z=0 in Definition 3.1 (i4) and (i5) and using (b1), we have

$$\mu_A(x) = \mu_A(x * 0) \geq \min\{\mu_A(x * (y * 0)), \mu_A(y)\}$$
$$= \min\{\mu_A(x * y), \mu_A(y)\}.$$

Also,

$$\gamma_A(x) = \gamma_A(x * 0) \leq \max\{\gamma_A(x * (y * 0)), \gamma_A(y)\}$$
$$= \max\{\gamma_A(x * y), \gamma_A(y)\}.$$

Hence (i2) and (i3) holds. Thus $A = (\mu_A, \gamma_A)$ is an Intuitionistic fuzzy ideal of X.

The converse of Proposition 3.3 is not true as seen the following example.

Example 3.4: Let $X = \{0, a, b, c\}$ be a BCI-algebra in which the $*$ - operation is given by the following table:

*	0	a	b	c
0	0	c	b	a
a	a	0	c	b
b	b	a	0	c
c	c	b	a	0

Note that {0} is an ideal of X, but not a q-ideal of X since $c * (0 * a) = c * c = 0 \in \{0\}$ by and $0 \in \{0\}$

But $c * a = b \notin \{0\}$. Define an Intuitionistic fuzzy subset

μ_A: X $\rightarrow [0,\ 1]$ and $\gamma_A : X \rightarrow [0, 1]$ by

$\mu_A = \begin{pmatrix} 0 & a & b & c \\ 0.8 & 0.7 & 0.5 & 0.5 \end{pmatrix}$ **and** $\gamma_A = \begin{pmatrix} 0 & a & b & c \\ 0.2 & 0.2 & 0.4 & 0.3 \end{pmatrix}$

Then $A = (\mu_A, \gamma_A)$ is an Intuitionistic fuzzy ideal of X, but not an Intuitionistic fuzzy q-ideal of X.

Corollary 3.5: If $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy ideal of a BCI-algebra X, then the following holds:

$(\forall x, y \in X)(x \leq y \ => (\mu_A(\mathrm{x}) \geq \mu_A(\mathrm{y}))$ and $(\gamma_A(\mathrm{x}) \leq \gamma_A(\mathrm{y}))$.

Proof: It follows from the Proposition 2.2 and Proposition 3.3.

Theorem 3.6: If $A = (\mu_A, \gamma_A)$ is an Intuitionistic fuzzy ideal of a BCI-algebra X, then the following are equivalent:

(1) $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy q-ideal of X,

(2) $(\forall\, x, y\ \in X)\ (\mu_A(x * y) \geq\ (\mu_A\big(x * (0 * y)\big)$ and $(\gamma_A(x * y) \leq\ (\gamma_A\big(x * (0 * y)\big)$
(3) $(\forall\, x, y, z\ \in X)\ (\mu_A((x * y) * z) \geq\ \mu_A\big(x * (y * z)\big)$ and $(\gamma_A((x * y) * z) \leq\ \gamma_A\big(x * (y * z)\big)$

Proof: (1) => (2).
Let $x, y \in X$. Putting $y = 0$ and $z = y$ in Definition 3.1 (i4), (i5) and use (d1), we have

$\mu_A(x * y) \geq$ min $\{(\mu_A(x * (0 * y), (\mu_A(0)\} = (\mu_A\big(x * (0 * y)\big)$ and

$\gamma_A(x * y) \leq$ max $\{(\gamma_A(x * (0 * y), (\gamma_A(0)\} = (\gamma_A\big(x * (0 * y)\big)$.

Thus (2) holds.
(2) => (3) Since for any $x, y, z \in X$,

$$\big((x * y) * (0 * z)\big) * \big(x * (y * z)\big) = \Big((x * y) * \big(x * (y * z)\big)\Big) * (0 * z)$$
$$\leq \big((y * z) * y\big) * (0 * z) = (0 * z) * (0 * z) = 0,$$

We have $(x * y) * (0 * z) \leq x * (y * z)$. Using (2) and Proposition 2.4, we get
$\mu_A(x * (y * z)) \leq \mu_A((x * y) * (0 * z) \leq\ \mu_A((x * y) * z)$ and
$\gamma_A(x * (y * z)) \geq \gamma_A((x * y) * (0 * z) \geq\ \gamma_A\big((x * y) * z\big)$.
Hence (3) holds.
(3) => (1) Let $x, y, z \in X$. Using (d2), (b2) and (3),we have

$$\mu_A(x * z) \geq \min\{\mu_A\big((x * z) * y\big), \mu_A(y)\}$$
$$= \min\{\mu_A\big((x * y) * z\big), \mu_A(y)\}$$
$$\geq min\ (\mu_A\big(x * (y * z)\big), \mu_A(y)\}.$$

Also,

$$\gamma_A(x * z) \leq max\ \{\gamma_A\big((x * z) * y\big), \gamma_A(y)\}$$
$$= \max\{\gamma_A\big((x * y) * z\big), \gamma_A(y)\}$$
$$\leq max\ (\gamma_A\big(x * (y * z)\big), \gamma_A(y)\}.$$

Hence $A = (\mu_A, \gamma_A)$ is an Intuitionistic fuzzy q-ideal of a BCI-algebra X.

Proposition 3.7: Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy ideal of X. If $\mu_A(\mathrm{x}) \leq \mu_A(x * y)$ and $\gamma_A(\mathrm{x}) \geq \gamma_A(x * y)$ for any $x, y, z \in X$, then A is an intuitionistic fuzzy q-ideal of X.
Proof : For any $x, y, z \in X$, we have

$$\mu_A(x * z) \geq \mu_A(x)$$
$$\geq min\ (\mu_A\big(x * (y * z)\big), \mu_A(y * z)\}$$
$$\geq min\ (\mu_A\big(x * (y * z)\big), \mu_A(y)\}.$$

Also,

$$\gamma_A(x * z) \leq \gamma_A(x)$$
$$\leq max\ (\gamma_A\big(x * (y * z)\big), \gamma_A(y * z)\}$$
$$\leq max\ (\gamma_A\big(x * (y * z)\big), \gamma_A(y)\}.$$

Hence $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy q-ideal of X.

4. Intuitionistic Fuzzy q-ideals of BCI-Algebras with degree in the inteval (0,1]

In what follows let X denote a BCI-algebra unless specified otherwise.

Definition 4.1 : Let I be a non-empty subset of a BCK/BCI- algebra X which is not necessary an ideal of X. We say that a subset J of X is an *enlarged ideal* of X related to I if it satisfies:
(1) I is a subset of J,
(2) $0 \in J$,
(3) $(\forall x \in X)(\forall y \in I)(x * y \in I\ => x \in J)$.

Definition 4.2: Let I be a non-empty subset of a BCI-algebra X which is not necessary a q-ideal of X. We say that a subset J of X is an *enlarged q-ideal* of X if it satisfies:
(1) I is a subset of J,

(2) $0 \in J$,

(3) $(\forall\, x, z \in X)(\forall y \in I)(x * (y * z) \in I \;=>\; x * z \in J)$

Obviously, every q-ideal is an enlarged q-ideal of X related to itself. Note that there exists an enlarged q-ideal of X related to any non-empty subset I of a BCI-algebra X.

Example 4.3: Let $X = \{0, 1, a, b, c\}$ be a BCI-algebra in which the $*$ operation is given by the following table:

*	0	1	a	b	c
0	0	0	a	b	c
1	1	0	a	b	c
a	a	a	0	c	b
b	b	b	c	0	a
c	c	c	b	a	0

Note that {0, a} is not both an ideal of X and a q-ideal of X. Then {0,1,a} is an enlarged ideal of X related to {0, a} and an enlarged q-ideal of X elated to {0, a}.

Theorem 4.4: Let I be a non-empty subset of a BCI-algebra X. Every enlarged q-ideal of X related to I is an enlarged ideal of X related to I.

Proof: Let J be an enlarged q-ideal of X related to I. Putting *z=0* in Definition 4.2(3) and use (b1), we have

$(\forall\, x \in X)(\forall y \in I)(x * (y * 0) \;=>\; x * y \in I \;=>\; x * 0 = x \in J)$.

Hence *J* is an enlarged ideal of X related to *I*.

The converse of Theorem 4.4 does not true in general as seen in the following example.

Example 4.5: Consider a BCI-algebra $X = \{0, a, b, c\}$ as in Example 3.4. Note that {0,a} is not both an ideal and a q-ideal of X. Then {0, a, b} is an enlarged ideal of X related to {0, a} but not an enlarged q-ideal of X related to {0, a} since $0 * (a * a) = 0 \in \{0, a\} and\ 0 * a = c \notin \{0, a, b\}$.

In what follows let $\alpha, \beta, \rho, and\ \sigma$ be members of (0, 1], and let *n* and *k* denote a natural number and a real number, respectively, such that *k* < *n* unless otherwise specified.

Definition 4.6: An intuitionistic fuzzy subset $A = (\mu_A, \gamma_A)$ of a BCK/BCI-algebra X is called an intuitionistic fuzzy ideal of X with degree $(\alpha, \beta, \rho, \sigma)$ if it satisfies:

(1) $(\forall x \in X)\big(\mu_A(0) \geq \alpha\, \mu_A(x)\big),\ \big(\gamma_A(0) \leq \beta\, \gamma_A(x)\big)$,

(2) $(\forall x, y \in X)(\mu_A(x) \geq \rho \min\{\mu_A(x * y), \mu_A(y)\})$,

(3) $(\forall x, y \in X)(\gamma_A(x) \leq \sigma\ \max\{\gamma_A(x * y), \gamma_A(y)\})$.

Definition 4.7: An intuitionistic fuzzy subset $A = (\mu_A, \gamma_A)$ of a BCI-algebra X is called an intuitionistic fuzzy q-ideal of X with degree $(\alpha, \beta, \rho, \sigma)$ if it satisfies:

(1) $(\forall x \in X)\big(\mu_A(0) \geq \alpha\, \mu_A(x)\big),\ \big(\gamma_A(0) \leq \beta\, \gamma_A(x)\big)$,

(2) $(\forall x, y, z \in X)(\mu_A(x * z) \geq \rho \min\{\mu_A(x * (y * z), \mu_A(y)\})$,

(3) $(\forall x, y, z \in X)(\gamma_A(x * z) \leq \sigma\ \max\{\gamma_A(x * (y * z)), \gamma_A(y)\})$.

Note that if $\alpha \neq \rho,\ \beta \neq \sigma$, then an intuitionistic fuzzy ideal of X with degree $(\alpha, \beta, \rho, \sigma)$ may not be an intuitionistic fuzzy q-ideal of X with degree $(\alpha, \beta, \rho, \sigma)$ and vice versa.

Proposition 4.8: If $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy q-ideal of a BCI-algebra X with $(\alpha, \beta, \rho, \sigma)$,
then A is an Intuitionistic fuzzy ideal of X with degree $(\alpha, \beta, \rho, \sigma)$.

Proof: Put *z=0* in Definition 4.7 (2) and (3).

Proposition 4.9: If $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy q-ideal of a BCI-algebra X with $(\alpha, \beta, \rho, \sigma)$, then

(1) $(\forall x, y \in X)\big(x \leq y \;=>\; \mu_A(x) \geq \alpha\rho\, \mu_A(y)\big),\ \big(\gamma_A(x) \leq \beta\sigma\, \gamma_A(y)\big)$.

(2) $(\forall\, x, y\ \in X)\ (\mu_A(x * y) \geq \alpha\rho\, \mu_A(x * (0 * y))$ and $(\gamma_A(x * y) \leq \beta\sigma\, \gamma_A(x * (0 * y))$

(3) $(\forall\, x, y, z\ \in X)\ (\mu_A\big((x * y) * z\big) \geq \alpha^2\rho^2\, \mu_A(x * (y * z))$ and $(\gamma_A((x * y) * z) \leq \beta^2\sigma^2\, \gamma_A(\big(x * (y * z)\big)$.

Proof: (1) Let $x, y\ \in X$ be such that $x \leq\ y$. Then $x * y = 0$. Putting $z = 0$ in Definition 4.7 (2) and (3) and using (b1), we have

$$\begin{aligned} \mu_A(x) = (\mu_A(x * 0) &\geq \rho \min\{\mu_A(x * (y * 0), \mu_A(y)\}) \\ &= \rho \min\{\mu_A(x * y), \mu_A(y)\} \\ &= \rho \min\{\mu_A(0), \mu_A(y)\} \\ &\geq \rho \min\{\alpha\mu_A(y), \mu_A(y)\} \\ &=\ \rho\ \alpha\mu_A(y). \end{aligned}$$

Also

$$\begin{aligned} \gamma_A(x) = (\ \gamma_A(x * 0) &\leq \sigma \max\{\gamma_A(x * (y * 0), \gamma_A(y)\}) \\ &= \sigma \max\{\gamma_A(x * y), \gamma_A(y)\} \\ &= \sigma \max\,\{\gamma_A(0), \gamma_A(y)\} \\ &\leq \sigma \max\{\beta\gamma_A(y), \gamma_A(y)\} \\ &=\ \sigma\ \beta\gamma_A(y). \end{aligned}$$

(2) Let $x, y\ \in X$. Putting $x = x, y = 0\ and\ z = y$ in Definition 4.7 (2) and (3), we have

$$\begin{aligned} \mu_A(x * y) &\geq \rho\ min\ \mu_A(x * (0 * y), \mu_A(0)) \\ &\geq \rho\ min\{\ \mu_A(x * (0 * y), \alpha\mu_A(\big(x * (0 * y)\big)\} \\ &= \alpha\rho\ \mu_A(\big(x * (0 * y)\big). \end{aligned}$$

Also,

$$\begin{aligned} \gamma_A(x * y) &\leq \sigma\ max\ \gamma_A(x * (0 * y), \gamma_A(0)) \\ &\leq \sigma\ max\{\ \gamma_A(x * (0 * y), \beta\gamma_A(\big(x * (0 * y)\big)\} \\ &= \beta\sigma\ \gamma_{\ A}(\big(x * (0 * y)\big). \end{aligned}$$

(3) Since

$$\begin{aligned} ((x * y) * (0 * z) * \big(x * (y * z)\big) &= ((x * y) * \big(x * (y * z)\big) * (0 * z) \\ &\leq \big((y * z) * y\big) * (0 * z) \\ &= (0 * z) * (0 * z) = 0, \forall x, y, z\ \in X, \end{aligned}$$

We get $(x * y) * (0 * z) \leq (x * (y * z))$.

It follows from (2) and Proposition 4.9(1) that

$$\begin{aligned} \mu_A\big((x * y) * z\big) &\geq \alpha\rho\ \mu_A((x * y) * (0 * z)) \\ &\geq \alpha^2\rho^2\mu_A\big(x * (y * z)\big). \end{aligned}$$

And

$$\begin{aligned} \gamma_A\big((x * y) * z\big) &\leq \beta\sigma\ \ \gamma_A((x * y) * (0 * z)) \\ &\leq \beta^2\sigma^2\gamma_A\big(x * (y * z)\big). \end{aligned}$$

Corollary 4.10: $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy q-ideal of a BCI-algebra X with $(\alpha, \beta, \rho, \sigma)$. If $\alpha = \rho, \beta = \sigma$, then the following hold:

(1) $(\forall\, x, y\ \in X)\big(x \leq y\ \;=>\; \mu_A(x) \geq \alpha^2\mu_A(y)\big),\ \gamma_A(x)\ \leq \beta^2\gamma_A(y))$.

(2) $(\forall\, x, y\ \in X)\ (\mu_A(x * y))\ \geq\ \alpha^2\mu_A(x * (0 * y))$ and $(\gamma_A(x * y))\ \leq\ \beta^2\gamma_A(x * (0 * y))$.

(3) $(\forall\, x, y\,, z \in X)\ (\mu_A((x * y) * z))\ \geq\ \alpha^4\mu_A(x * (y * z))$ and $(\gamma_A((x * y) * z)) \leq \beta^4\gamma_A(x * (0 * y))$.

Proposition 4.11: Let $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy ideal of a BCI-algebra X with $(\alpha, \beta, \rho, \sigma)$. If $\mu_A(x) \leq \mu_A(x * y), \gamma_A(x) \geq \gamma_A(x * y)$ for any $x, y\ \in X$, then A is an intuitionistic fuzzy q-ideal of X with degree $(\alpha, \beta, \rho, \sigma)$.

Proof: For any $x, y, z \in X$, we have

$$(\mu_A(x)) \geq \rho \min \{\mu_A(x * (y * z)), \mu_A(y * z)\}),$$
$$(\gamma_A(x)) \leq \sigma \max \{\gamma_A(x * (y * z)), \gamma_A(y * z)\}).$$

By assumption, we obtain $\mu_A(x * z) \geq \mu_A(x))$

$$\geq \rho \min \{\mu_A(x * (y * z)), \mu_A(y * z)\}$$
$$\geq \rho \min \{\mu_A(x * (y * z)), \mu_A(y)\}$$

and $\gamma_A(x * z) \leq \gamma_A(x))$

$$\leq \sigma \max \{\gamma_A(x * (y * z)), \gamma_A(y * z)\}$$
$$\leq \sigma \max \{\gamma_A(x * (y * z)), \gamma_A(y)\}$$

Thus A is an Intuitionistic fuzzy q-ideal of X.

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